

5.2 & (Marginal and conditional pdf)

(Marginal)

$$: P(x_1) = \sum_{x_2} P(x_1, x_2) \quad f(x_1) = \int f(x_1, x_2) dx_2$$

$$: P(x_1 | x_2) = \frac{P(x_1, x_2)}{P(x_2)}, f(x_1 | x_2) = \frac{f(x_1, x_2)}{f(x_2)} \quad : P(A|B) = \frac{P(A, B)}{P(B)}$$

가 .

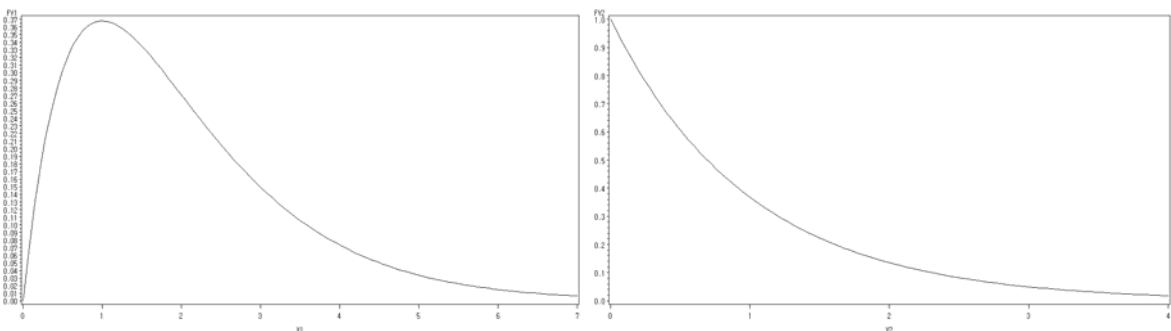
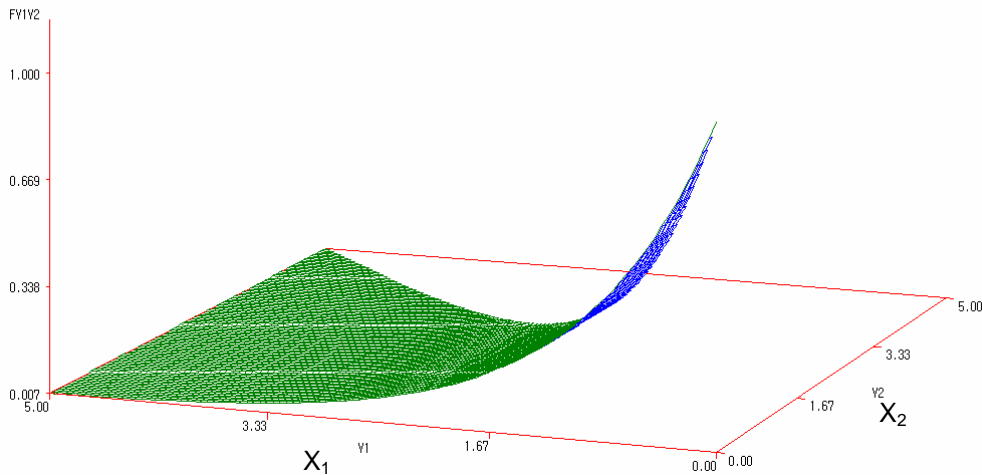
$$P(x_1) = \sum_{x_2} \dots \sum_{x_n} P(x_1, x_2, \dots, x_n)$$

$$f(x_1) = \int \dots \int f(x_1, x_2, \dots, x_n) dx_2 dx_3 \dots dx_n \quad : f_{x_1}(x_1) = f_1(x_1) = f(x_1)$$



EXAMPLE 5.7

$$f(x_1, x_2) = e^{-x_1}, 0 \leq x_2 \leq x_1 < \infty \quad \text{pdf}$$





EXAMPLE 5.8

Example 5.1 ()

X_2 가 ?

? $P(x_1 | X_2 = 1)$

	x_1	0	1	2
x_2		0	1	2
0		1/9	2/9	1/9
1		2/9	2/9	0
2		1/9	0	0



EXAMPLE 5.9

Example 5.4, $f(x_1, x_2) = 3x_1, 0 \leq x_2 \leq x_1 \leq 1$ ()

X_1 가 X_2 가

X_2 가 X_1 가

$f(x_1 | x_2)$ 가

(conditional density function)

$X_2 = x_2$ 가

X_1 가

$F(x_1 | x_2) = P(X_1 \leq x_1 | X_2 = x_2)$

$f(x_1 | x_2)$ 가



EXAMPLE 5.10

가 $f(x_1, x_2) = 1/2, 0 \leq x_1 \leq x_2; 0 \leq x_2 \leq 2$

$P(X_1 \leq 1/2 | X_2 = 2)$?

$f(x_2) = x_2 / 2, 1/4$

**EXAMPLE 5.11**

$$\text{Find } f(x_1, x_2) = 2, 0 \leq x_1 + x_2 \leq 1; 0 \leq x_1, x_2 \leq 1 \quad P(X_1 \geq 1/2 | X_2 \leq 1/4) ?$$

**HOMEWORK #12-4**

DUE 5 10 ()

The joint pdf of (X_1, X_2) is $f(x_1, x_2) = x_1 + x_2, 0 \leq x_1 \leq 1; 0 \leq x_2 \leq 1$.

Find the marginal pdf of X_1 and X_2 .

Find $P(X_1 \geq 1/2 | X_2 \geq 1/2)$.

Find $P(X_1 \geq 3/4 | X_2 = 1/2)$

**HOMEWORK #12-5**

DUE 5 10 ()

The joint pdf of X_1, X_2 is $f(x_1, x_2) = 6(1 - x_2), 0 \leq x_1 \leq x_2 \leq 1$.

Find the marginal pdf of X_1 .

Find the conditional pdf, $f(x_2 | x_1)$.

Find $P(X_2 \leq 1/2 | X_1 \leq 3/4)$.

Find $P(X_2 > 3/4 | X_1 = 1/2)$.

**HOMEWORK #12-6**

DUE 5 10 ()

The joint pdf of X_1, X_2 is $p(x_1, x_2) = \frac{\binom{4}{x_1} \binom{3}{x_2} \binom{2}{3-x_1-x_2}}{\binom{9}{3}}, 0 \leq x_1, x_2 \leq 3, 1 \leq x_1 + x_2 \leq 3$.

Find the marginal pdf of X_1 . Is it a popular distribution?

Find $P(X_2 = 1 | X_1 = 2)$.

5.3 Independence ()

$$P(AB) = P(A)P(B) .$$

(independence)

$$\begin{aligned}
 & f(x_1, x_2) = f_1(x_1)f_2(x_2) \quad (\quad F(x_1, x_2) = F_1(x_1)F_2(x_2) \quad) \quad X_1 \\
 X_2 & \quad \cdot \quad X_1 \quad X_2 \quad f(x_1, x_2) = f_1(x_1)f_2(x_2) \\
 \cdot X_1, X_2 & \quad \Leftrightarrow (\text{iif}) f(x_1, x_2) = f_1(x_1)f_2(x_2)
 \end{aligned}$$

(Random Sample)

$$X_1, X_2, \dots, X_n \quad \Leftrightarrow (\text{iif}) X_i \underset{\sim}{iid} f(x)$$

IID: Independently() and Identically distributed()

$$f(x_1, x_1, \dots, x_n) = (\text{independently}) f(x_1)f(x_2)\dots f(x_n) = (\text{identically}) [f(x)]^n$$



EXAMPLE 5.12

Example 5.1, X_1, X_2 가? No



EXAMPLE 5.13

$f(x_1, x_2) = 6x_1x_2, 0 \leq x_1, x_2 \leq 1$ X_1, X_2 가? Yes



EXAMPLE 5.14

$f(x_1, x_2) = 2, 0 \leq x_2 \leq x_1 \leq 1$ X_1, X_2 가? No

(Theorem)

X_1, X_2 (range)가
 $f(x_1, x_2) = g(x_1)h(x_2),$

..

**EXAMPLE 5.15**

$$f(y_1, y_2) = 2y_1, 0 \leq y_1, y_2 \leq 1$$

 X_1, X_2

가?

Yes

**HOMEWORK #13-1**

DUE 5 12 ()

The length of life X for a bulb is distributed as $f(x) = \frac{1}{3}e^{-x/3}, 0 < x$. Randomly selected two bulbs have independently length of X_1, X_2 . Find $P(X_1 + X_2 \leq 1)$.

**HOMEWORK #13-2**

DUE 5 12 ()

Two telephone calls come into a switchboard at random times in a fixed 1-hour period. Assume that the calls are made independently of one another.

What is the probability that both calls are made in the first half hour?

What is the probability that the calls are made within 5 minutes of each other?